Market equilibrium with management costs and implications for insurance accounting *

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Abstract

The valuation approach used in practice under Solvency II and other insurance regulatory accounting regimes, incorporates investment management costs, while ignoring that other market participants also incur such costs. We show within a general equilibrium framework that a correction term representing the market's average management costs is missing in regulatory valuations.

For insurers subject to Solvency II, we estimate the correction is of the order of €150 billion representing 2% of investments or 16% of own funds.

Current practice distorts incentives against expensive to manage assets, which are typically assets policyholders cannot access directly.

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1. Introduction

The insurance industry manages a large asset base dominated by investments backing long duration pension savings. The European Economic Area (EEA) insurers subject to Solvency II manage € $8,500$ bn of investments.[1] Insurers manage a large part of the financial wealth of private households. For example according to FFA (2021) and GDV (2020), French and German households respectively hold 38% and 22% of their financial wealth via insurance contracts. Given the size of investments under management insurers obviously also play an important role in the allocation of capital throughout the economy.

In the past years insurance regulation has been moving globally to a risk based capital approach based on valuation methodologies developed in academic literature, namely arbitrage free and market consistent pricing. The Swiss regulator was one of the early adopters via the Swiss Solvency Test (SST) in 2008, followed by the European Union with Solvency II applicable in the EEA since 2016. Bermuda, a global insurance hub, implemented the Bermuda Solvency Capital requirement (BSCR) which is considered equivalent to Solvency II.

In practice, valuation of market and investment related risks is implemented via (risk neutral) pricing probabilities, which were first introduced in a general equilibrium setting by Drèze (1970) and further developed notably by Ross (1977) and Cox and Ross (1976). Regulation makes explicit that investment management expenses need to be factored into this cashflow valuation.[2] In practice, not only the directly incurred such as salaries for employees managing investments, the cost for the infrastructure for these employees such as office space and software are considered, but also outsourced costs such as fees paid

1. Source: Eiopa insurance statistics. Numbers are from 2020 including the UK. In addition those insurers also manage € $3,300$ bn of unit-linked investments on behalf of policyholders.

2. See, e.g. EU Parliament (2015) §31.1: "A cash flow projection used to calculate best estimates shall take into account […] investment management expenses".

to external fund managers and custody fees which are typically deducted from the funds net asset value EIOPA (2021).

The widely, albeit not unanimously, used approach, consists in valuing investment related cashflows, assuming at least implicitly a financial market without investment management costs, and then subtracting the present value of the considered companies' expected future management costs under the same pricing measure. In fact, this approach is already documented common practice in CFO-Forum (2009)[3] for the calculation of shareholder value in a life insurance book under the market consistent embedded value approach which was introduced by a group of European insurance CFOs. Management costs of market participants other than the company are not taken into consideration, which leads to an internal contradiction[4] of the Solvency II model. Our analysis shows that the standard method leads to widespread double counting of costs.

If all investors would incur the same management costs, it would be simple to correct the error in the current valuation approach. One would simply need to stop adding the present value of own management costs to the liability side, as they are already contained in market prices. As the regulation seems to impose factoring in the individual management costs, we need a theory of financial markets and valuation with heterogeneous management costs in order to develop a consistent valuation approach.[5]

3. Compare to page 27, paragraph 137 of principle 13. Embedded value balance sheets were published by many listed European insurers before the introduction of Solvency 2. The embedded value should give a measure of the value form a shareholder perspective of a life insurance balance sheet, which mainly consists of a risk adjusted computation of discounted future cashflows paid to the shareholder. For complex life insurance products, this is typically based on risk neutral Monte Carlo simulation techniques, similar to the valuation techniques used for exotic derivatives

4. Insurers when constructing their Solvency II balance sheet value their costs, but implicitly assume that all other market participants including other insurers subject to Solvency II incur no costs.

5. Although there is an important academic literature on transaction costs (see, e.g. Jouini and Kallal 1995; Cvitanić et al. 1999; Czichowsky et al. 2018), investment management costs have not received the same attention.

We study a financial economy where investors incur investment management costs, which can differ from one investor to another. Since market consistent valuation of a portfolio relies on observed market prices, our first focus is to study how management costs impact equilibrium prices. We show that, compared to a situation without costs, their presence deflates equilibrium prices by a factor measured by a weighted average of market participant costs. The weights associated to each market participant factor both their elasticities of demand for the asset and the size of their market position. We then derive a valuation formula for cashflows, factoring-in the cost structure of the company holding that portfolio. This formula deducts the company's own costs from the market value of the assets generating the cashflow, and it adds back the weighted average of the market's management costs as a correction term.

The correction term can be meaningful for (a) life insurers with investments backing long term liabilities, (b) insurers investing in complex to manage investments, and especially for insurers combining both aspects. Giving access to such assets via a pooled investment process is arguably one of the potential value propositions of life insurers. The status quo valuation approach not only introduces an overestimated impact of investment management costs on own funds, but among other potential unintended consequences it may distort investment strategies by incentivising insurers to overweight assets that are relatively cheap to manage.

In section 2 we introduce the financial model, in section 3 we establish valuation formulas, starting with the approach typically used to comply with the regulation. In section 4 we study risk neutral probabilities in the presence of investment management costs. Section 5 estimates the potential impact of our valuation approach on Solvency II balance sheets. We conclude in section 6.

2. Market equilibrium with management costs

We study a standard two-period exchange economy with a safe asset and a risky asset, as in, e.g. Fishburn and Burr Porter (1976), Dana (1995) and Gollier (2001). We compare prices of the risky asset between the economy in which (a) these assets bear management costs to its holder, (b) to those prices in an economy without such costs. We then interpret the difference between those two prices as the measure of the extent to which management costs are factored into observed market prices.

The finite set of agents is denoted I, the time periods are $t = 0, 1$, the economy has a safe asset, paying interest rate ρ between period 0 and 1, and a risky asset, that pays a random amount x with cumulative distribution function F at period 1. We assume that the risky asset does not have negative payoff, $F(0) = 0$, and has finite expectation, $\mathbf{E}\, x < \infty$.

Each agent i has an initial endowment $w_{ic} \geq 0$ in the riskless asset and $w_{ia} \geq 0$ in the risky asset. These assets are traded at $t = 0$, payoffs are realized at $t = 1$, and i's von-Neumann Morgenstern's utility function in numéraire is denoted U_i, where

$$U_i : \mathbb{R}_+ \to \mathbb{R} \cup \{-\infty\}$$

with $u_i(z) \in \mathbb{R}$ for all $z > 0$. We assume U_i to be twice differentiable, $U_i' > 0$ and $U_i'' < 0$. The Arrow-Pratt coefficient of relative risk aversion is defined as $R_i^r(w) = -xU_i''(w)/U_i'(w)$. Agent i incurs a management cost of $m_i \geq 0$ for each unit of the risky asset, so the net payoff of the risky asset to agent i is $x - m_i$. As a survival assumption, we will assume that there exists $\varepsilon > 0$, such that for all i, $x - m_i \geq \varepsilon$ almost surely.

Given a unit price $p > 0$ and a quantity a in the risky asset, i's indirect utility function is,

$$V_i(a, p, m_i) = \mathbf{E}\, U_i((1 + \rho)w_{i0} + a(x - m_i - (1 + \rho)p)) \tag{2.1}$$

where $w_{i0} = w_{ic} + pw_{ia}$ is i's initial wealth. Agent i's demand in the risky asset is:

$$D_i(p, m_i) = \arg\max_a V_i(a, p, m_i)$$

where the maximum is taken over all $a \geq 0$ that satisfy the budget constraint $pa \leq w_{i0}$.

Since V_i is concave in a, $D_i(p, m_i)$ is well defined and unique for every m_i and p.

Useful properties of D_i are summarized below.

Proposition 1. *Assume that i's relative risk aversion is bounded by 1. Then*

1. $D_i(p, m_i) = 0$ *if* $p \geq \frac{\mathbf{E}x - m_i}{1+\rho}$ *and* $D_i(p, m_i) > 0$ *otherwise,*
2. *the left-hand and right-hand derivatives of* D_i *exist at all* $p > 0$,
3. $D_i(p, m_i)$ *is decreasing in* p *for* $p < \frac{\mathbf{E}x - m_i}{1+\rho}$,
4. $\lim_{p \to 0} D_i(p, m_i) = \infty$.

Relative risk aversion bounded by 1 in several cases of interest, including logarithmic utility functions. It is also consistent with Chetty (2006)'s estimate of 0.71 for the mean relative risk aversion in the population. In the rest of the paper, we assume that the conclusions of Proposition 1 hold, which subsumes, but is not limited to the case of relative risk aversion bounded by 1.

The total demand when management costs for all agents are represented by a vector $m = (m_i)_i$ is given by $D(p, m) = \sum_i D_i(p, m_i)$. The total supply of risky asset in the economy is $W_a = \sum_i w_{i,a}$. An equilibrium price is a price p such that the total demand and supply equalize, it thus satisfies:

$$D(p, m) = W_a. \tag{2.2}$$

We have established that D is continuous, equal to 0 for p large enough and going to ∞ for $p \to 0$. Locally, it is either equal to 0 (when $p \geq \frac{\mathbf{E}x - m_i}{1+\rho}$ for all i) or decreasing in p. By the intermediate value Theorem, there is a unique value of p such that $D(p; m) = w_a$, therefore for every vector m of management costs, there exists a unique equilibrium price $p^*(m)$.

3. Valuation with management costs

Under Solvency II, SST and BSCR regulations, insurers are required to value assets and liabilities at fair value.[6] Traditional life insurance liabilities depend on cash flows of the investments which cover these liabilities. To value those liabilities the Solvency II delegate regulation EU Parliament (2015) requires insurers to use an arbitrage free and market consistent model[7] and moreover to consider for their valuation their own management expenses,[8] Similar requirements exist for SST in FINMA (2020) and BSCR in Bermuda Monetary Authority (2011).[9]

In practice insurers implement those requirements via a (potentially stochastic) projection of asset and liability cashflows and discounting them with a risk free interest rate curve. The discounting process is constructed so that the gross discounted investment cashflows correspond to the market value of the investments. Hence, those assets are valued assuming implicitly a market where all other investors bear no management costs. In parallel, the present value of the future investment management costs are added to the liability side.

6. See EU Parliament (2009), (45), (54), § 75, 76.3 and EU Parliament (2015) §10 for Solvency II.

7. See e.g. §22.3: "Where insurance and reinsurance undertakings use a model to produce projections of future financial market parameters, it shall comply with all of the following requirements: (a) it generates asset prices that are consistent with asset prices observed in financial markets; (b) it assumes no arbitrage opportunity".

8. See §31: "A cash flow projection used to calculate best estimates [...] takes into account various expenses including investment management expenses".

9. See pages 3, 13 and 20 for FINMA (2020) and page 152 paragraph 8.$(b)(ii)$ and paragraph 15 for Bermuda Monetary Authority (2011). We focus here on capital metrics according to the strictest metrics, in terms of Solvency II the Solvency II balance sheet without transitional measures, volatility and matching adjustment, or in terms of BSCR assuming the equivalent so-called scenario based approach.

Here insurance companies use costs they are expected to incur for managing the investments they plan to hold to back the liabilities.

To translate this into our model, let us consider insurer i investing in a unit of the risky asset to back insurance liabilities. The asset cashflow minus the costs m_i to manage this investment is valued as

$$v_i \quad = \quad p(0) - \frac{m_i}{1+\rho} \tag{3.1}$$

where $p(0)$ represents the economic value of a unit of the asset in an economy where agents incur no management costs. However, given that not only the insurance company (which might be small compared to the wider market) incurs investment management costs, but all other insurance companies and more generally all other investors incur investment management costs, $p(0)$ is of course not observed. Only $p(m)$ is observed on the market. The distinction between observed prices $p(m)$ and prices absent of management costs $p(0)$ is ignored by the insurers who use $p(m)$ as if it where $p(0)$ in their models.

We show below that the current approach is omitting a correction term, leading to some double counting of costs. For this we start by examining $p(m) - p(0)$. Overall investment management costs reported by institutional investors at a portfolio level are typically relatively small of the order of $10 - 20$bps (depending on the complexity of the asset classes chosen of course). We will use the first order approximation

$$p(m) - p(0) \sim \sum_i \left(\frac{\partial p^*}{\partial m_i} m_i \right) \tag{3.2}$$

To understand how management costs impact equilibrium prices p, we first study their impact on demand. Note that managements costs impact demand through both directly, since they impact net cashflows from the risky asset, and indirectly, through wealth effects. In fact, since management costs impact prices, they also impact agents' initial wealth $w_{i0} = w_{ic} + pw_{ia}$, hence their attitude towards risk. For simplification of exposition, we first analyze the case in which agents' preferences exhibit constant absolute risk aversion (CARA), in which case wealth effects are absent. We then show that, in the more general and more realistic case that agents exhibit decreasing absolute risk-aversion (DARA), the impact of management cost on demand - and on prices - can only be larger than in the CARA case.

To make the wealth effects explicit, we let $D_i(w, p; m_i)$ be the demand from agent i with initial wealth w. With CARA preferences, $D_i(w, p; m_i)$ is independent of w, whereas with DARA preferences it is non-decreasing in w (cf Arrow (1965); Pratt (1964)). We then have $D_i(p, m_i) = D_i(w_{ic} + pw_{ia}, p, m_i)$.

We note that agent i's final wealth can be rewritten:

$$(1 + \rho)w_{i0} + a(x - m_i - (1 + \rho)p)) = (1 + \rho)w_{i0} + a(x - (1 + \rho)p')) \tag{3.3}$$

with $p' = p + \frac{m_i}{1+\rho}$ and $w_{i0} = w_{ic} + pw_{ia} = w_{ic} + p'w_{ia} - \frac{m_i}{1+\rho}w_{ia}$. If follows that

$$D_i(w_0, p, m_i) = D_i(w_0 - \frac{m_i}{1 + \rho}w_{ia}, p + \frac{m_i}{1 + \rho}, 0). \tag{3.4}$$

The above equation allows to compare demand with and absent of management costs. It has a natural interpretation. Management costs make the risky asset more expensive by $\frac{m_i}{1+\rho}$: this is the direct effect. This effect on price makes the agent means that in order to keep initial wealth constant we need to deflate their initial wealth by $\frac{m_i}{1+\rho}w_{ia}$: these are wealth effects.

3.1. Analysis absent wealth effects

Now we estimate the impact of (small) management costs on the equilibrium price p^*, around $m_i = 0$, when the agent has CARA preferences. In this case, equation (3.4) becomes:

$$D_i(p, m_i) = D_i(p + \frac{m_i}{1 + \rho}, 0) \tag{3.5}$$

The elasticity of demand for agent i is given by

$$e_i(p;m) = -\frac{\partial \log D_i}{\partial \log p} = -\frac{p}{D_i}\frac{\partial D_i}{\partial p}$$

where all derivatives are taken to be right-hand derivatives.

Differentiating (3.5) wrt. m_i at $m_i = 0$ gives:

$$\frac{\partial D_i}{\partial m_i}(p;0) \quad = \quad \frac{1}{1+\rho}\frac{\partial D_i}{\partial p}(p;0) = -\frac{D_i}{p(1+\rho)}e_i, \tag{3.6}$$

where here again $\frac{\partial D_i}{\partial m_i}$ is a right-hand derivative, and $\frac{\partial D_i}{\partial p}$ a left-hand one.

By differentiating (2.2) and applying the implicit function Theorem we obtain:

$$\frac{\partial p^*}{\partial m_i} = -\frac{\frac{\partial D_i}{\partial m_i}}{\sum_j \frac{\partial D_j}{\partial p}} \tag{3.7}$$

where $\frac{\partial p^*}{\partial m_i}$ and $\frac{\partial D_i}{\partial m_i}$ are taken as the right-hand derivatives, and $\frac{\partial D_j}{\partial p}$ as left-hand derivatives (since an increase in m_i leads to a decrease D for constant p, which is compensated by an decrease of p).

Finally, we combine (3.7) and (3.6) and obtain:

$$\frac{\partial p^*}{\partial m_i} = -\frac{1}{1+\rho}\frac{D_i e_i}{\sum_j D_j e_j}. \tag{3.8}$$

Using (3.8), we obtain the fundamental expression

$$p(0) \sim p(m) + \frac{1}{1+\rho}\frac{\sum_i D_i e_i m_i}{\sum_j D_i e_i}. \tag{3.9}$$

We let the *cost correction term* be:

$$\bar{c}(m) = \frac{\sum_i D_i e_i m_i}{\sum_j D_i e_i},$$

and we can now rewrite (3.1) to obtain the alternative expression:

$$v_i \quad = \quad p(m) + \frac{1}{1+\rho}\left(\bar{c}(m) - m_i\right) \tag{3.10}$$

The price $p(m)$ is the observed market price. The cost correction term is the average of agents' costs weighted by their portfolio sizes D_i, costs m_i and elasticities e_i. Demands D_i and associated costs m_i can to some extent be inferred from financial reporting.[10] Elasticities e_i are not directly observed, but may either be econometrically estimated or inferred through agents' risk attitudes. If we make the simplifying assumption that all agents have the same price elasticity in the risky asset we obtain that:

$$\bar{c}(m) = \frac{\sum_k D_k m_k}{\sum_k D_k}, \tag{3.11}$$

hence the cost correction term is the average of costs weighted by portfolio sizes, which can be estimated from reported data.

The difference of management costs incurred by reasonably efficient investors should be an order of magnitude smaller than the average management costs. The difference might be driven by differences in efficiency, or differences in strategy or quality in managing the assets. This means that $\bar{c}(m) - m_i$ should be relatively small compared to $c(m)$ itself.

Hence, for an insurer who has no evidence that they are excessively more or less efficient than the market average, it might be reasonable to assume that their costs m_i are the same for all market participants, in which case $m_i = \bar{c}(m)$ for every i independently of the elasticities and portfolio sizes. The valuation of equation (3.10) is simplified to

$$v_i = p(m) \tag{3.12}$$

and hence in such a case investment management costs cancel out.

3.2. Wealth effects

We now analyze the more general case in which U_i exhibits decreasing absolute risk aversion (DARA). In this case, demand for the risky asset is non-decreasing

10. However, the management costs modeled in the regulatory balance sheet, which may differ from management costs reported in the financial statements are typically not reported.

in the agent's initial wealth:

$$\frac{\partial D_i}{\partial w_0}(w_0, p, m_i) \geq 0. \tag{3.13}$$

Differentiating (3.4) wrt. m_i gives:

$$\begin{aligned}\frac{\partial D_i}{\partial m_i}(w_0, p, m_i) &= -\frac{w_{ia}}{1+\rho}\frac{\partial D_i}{\partial w_0}(w_0, p, m_i) - \frac{D_i e_i}{p(1+\rho)} \\ &\leq -\frac{D_i e_i}{p(1+\rho)}\end{aligned}$$

Now, by combining this last inequality with (3.2) we obtain

$$p(0) \geq p(m) + \frac{1}{1+\rho}\frac{\sum_i D_i e_i m_i}{\sum_j D_i e_i}$$

and therefore

$$v_i \quad \geq \quad p(m) + \frac{1}{1+\rho}\left(\bar{c}(m) - m_i\right) \tag{3.14}$$

meaning that the correction term obtained absent of wealth effects is a lower bound of its true value if wealth effects are taken into account.

This has a natural economic interpretation. In fact, by the direct effect, an increase in management costs make the risky asset less attractive, which in turn reduces its price. Moreover, by the wealth effect, when the price of the risky asset goes down, the agent's initial wealth goes down as well, and so its aversion to risk increases, leading to subsequently lower demand for the risky asset, lowering its price again. As we see, wealth effects amplify the direct effect, hence lead to a larger correction term than absent wealth effects.

4. Risk-neutral probabilities

According to EIOPA (2019a) §3.3.93, *"for valuing the best estimate for non-unconditional benefits, a stochastic simulation approach would consist of an appropriate market consistent asset model for projections of risk-neutral returns"*. It is therefore important to understand the implications of risk-neutral pricing onto our model. Following Drèze (1970), Ross (1977) and Cox and Ross (1976), risk-neutral probabilities [11] are probabilities Q over states of nature such that the price of every security X with payoff $X(s)$ in state s can be computed as

$$p_X = \frac{1}{1+\rho} \int_s X(s) dQ(s). \tag{4.1}$$

Risk-neutral probabilities exist under absence of arbitrage opportunity. They are unique if furthermore markets are complete. It should be noted that these probabilities *do not* represent objective probabilities of events, like probabilities over coin flips, but merely a convenient pricing instrument.

It follows from (4.1) that, under the risk-neutral probability asset, the return every asset is the same in every state, and is equal to the risk-free return.

In our economy, each state s is associated with a payoff from the risky asset. There are two assets: the risk-free one which pays $1+\rho$ in every state of s, and the risky asset, which pays $X(s)$ in state s (where the state space S, P is the underlying probability space).

Since markets are not necessarily complete, the risk-neutral probability measures are not necessarily unique. Any probability distribution Q that satisfies (4.1) for both the risk-less and the risky assets is a risk-neutral probability.

Since the price $p(m)$ of the risky asset depends on management costs m, so do the corresponding risk-neutral probabilities. Therefore, an appropriate computation of risk-neutral probabilities should take into account cost considerations. If we denote by Q^m a risk-neutral probability when management costs are m, we have

$$p(m) = \frac{1}{1+\rho} \mathbf{E}_{Q^m} X \tag{4.2}$$

11. We consider here risk-neutral probabilities using the risk-free asset as numéraire, but the discussion extends naturally to other reference asset.

and in particular

$$p(0) = \frac{1}{1+\rho}\mathbf{E}_{Q^0}X \tag{4.3}$$

In practice, the relationships (4.1) are used to derive risk-neutral probabilities, under which classes of assets can be priced. Note that here we presented what we can call *gross cashflow risk neutral probabilities.* They value the asset discounting the gross cashflows. Especially when all costs are equal, it would be more convenient to work directly on net cashflows which would lead to a different pricing probability, which one might call *net cashflow risk neutral probabilities.* When there are no management costs both concepts coincide.

To follow a valuation approach starting with an assumption of zero costs require an evaluation of Q^0. In turn, the evaluation of Q^0 requires an prior estimation of $p(0)$, namely of the prices of assets absent management costs.

In order to properly calibrate a model of risk-neutral probabilities, one can therefore

1. Estimate $p(0)$ from observed market prices and using formula (3.9),
2. calibrate risk-neutral probabilities $Q(0)$ from thus obtained prices,
3. value portfolios under $Q(0)$,
4. subtract discounted management expenses from the obtained value, thus obtaining the net value v_i of the portfolio.

We then obtain

$$v_i = \frac{1}{1+\rho}\mathbf{E}_{Q(0)}X - \frac{m_i}{1+\rho} = p(0) - \frac{m_i}{1+\rho} \tag{4.4}$$

which is the same as formula (3.1). It follows that v_i obtained using risk-neutral probabilities indeed coincides with formula (3.10). This concludes that our pricing method is in fact the same as under risk-neutral probabilities, once these probabilities are properly derived from the price system.

As an alternative to the method above, each firm can compute risk-neutral probabilities by equating asset prices with expected gross returns, thus using

Q^m instead of Q^0 for the management of their assets. The advantage of this method is that Q^m is directly inferred from market prices, whereas Q^0 is not. If firms value portfolios based on Q^m, and subtract discounted management expenses from the obtained value, they need to also add back the discounted value of the cost correction term $\bar{c}(m)$ to asset prices. In fact, the relationship

$$\mathbf{E}_{Q^m} X - \frac{m_i}{1+\rho} + \frac{\bar{c}(m)}{1+\rho} = \mathbf{E}_{Q^0} X - \frac{m_i}{1+\rho} = p(0) - \frac{m_i}{1+\rho}$$

shows that the pricing thus obtained coincides with (3.1).

5. Impact estimate

As the exact cost modeling of insurers is not public, any impact estimation is necessarily very imprecise. Costs vary massively with the investment strategy as may be illustrated from industry data. For example, CEM Benchmarking surveys the cost structure of pension funds. The below table is based on European pension funds with more than € $2,000$ bn of assets under management, as reported by Beath and Flynn (2018).

Asset class	Dutch	other EU	UK
Public equity	7	12	11
Private equity	454	382	415
Fixed income	6	4	5
Hedge funds	261	258	227
Listed real estate	28	24	78
Unlisted real estate	114	46	69
Infrastructure	159	150	187
Other	31	64	100

Table 1. CEM benchmarking 2018. Costs in bps

For a set of US pension funds with \$ 2900 bn of assets under management, Beath and Flynn (2020) find costs for US fixed income ranging from 9 bps (other) to 18 bps (long duration bonds). For Europe, we are not aware of any publicly available survey with granular data for management costs by fixed income subclasses. Furthermore costs of back/middle office and more general staff not dedicated to a single asset class may need to be considered as well. Ambachtsheer (2018) reports costs of 1.5bps for internal oversight functions.

Even if we cannot estimate precisely the impact of the proposed method compared to the erroneous approach of cost double counting ignoring the correction term, we can illustrate what is at stake. We do this by comparing our approach to an approach where current reported costs relative to the size of the investments are assumed to persist over the life of the liabilities, and we use a simple, back of the envelope approach.

As we have shown in our model, the difference in asset prices between the two methods is measured by the discounted value of the correction term. In a model with more than two periods, this discounted value can be obtained using the liability duration, Du. Hence a price correction of $Du \times c(m)$. In order to obtain the monetary value S of the correction, one needs further to multiply by total portfolio value I. This leads to the simple estimate for the monetary impact as:

$$S = I \times Du \times \bar{c}.$$

Table 2 presents data on reported investments, investment management costs and the liability duration by country, taken from EIOPA's insurance statistics and EIOPA (2019b).[12] From this we deduce estimations shown for our correction term relative to the investments as well as in absolute € amounts.

The costs reported under statutory accounting do not contain costs charged directly to the net asset value of collective investment undertakings (CIU). To estimate overall costs, we assumed that costs charged to funds are in line with the statutory costs which cover mainly the investments directly on balance sheet. Costs related to private equity investments booked under *Holdings in related undertakings, including participations* should not be included in

12. See page 38.

reported statutory costs, which might require a further adjustment. Indeed, it is expected by regulators to model costs incurred at fund level.

Country	I € bn	SC € bn	H	CIU	C bps	Du	$Du \times C$	S € bn
DE	2,260	2.4	19%	31%	15	19.4	3.0%	67
DK	276	0.6	28%	23%	27	14.1	3.8%	10
FR	2,357	1.9	7%	19%	10	11.8	1.2%	27
IT	821	0.8	11%	13%	11	9	1.0%	8
NL	388	0.4	6%	8%	10	13.4	1.4%	5
UK	1,062	2.5	14%	9%	26	9.6	2.5%	26
EEA	8,139	11.1	13%	20%	17	11.9	2.0%	165

TABLE 2. 2020 investments and costs, I=investments-derivatives+loans and mortgages, SC=statutory costs, H=Holdings in related undertakings, including participations, CIU=collective investment undertakings, $C = \frac{SC}{I\times(1-CIU)}$ costs adjusted for costs not reported for CIU, Du=liability duration $Du \times C$=correction term in percentage points of investments $S = I \cdot Du \cdot C$ absolute correction term or stake vs. naive modeling approach

As can be seen in table 2, the total impact on EEA is estimated at an order of magnitude of € 165 bn, with most impacted countries being Germany (€ 67 bn) followed by France (€ 27 bn) and UK (€ 26 bn).

These impact measures should be understood as between our method compared to the strictest approach where all costs are modeled gross. Also, the impact is considered with respect to the strictest Solvency II metric, without any transitional measures, or other permanent measures as volatility or matching adjustment. In a recent consultation paper, EIOPA (2021)[13] suggests that all expenses should be taken into account in line with the strategy at least for investments backing technical provisions and investments backing the solvency requirement. Hence, only management costs for investments backing excess capital could be ignored, if those considerations are implemented. Also, at least in some countries it seems common practice not to include costs related to real estate investments. These imply that the real effect of an implementation of our correction term would be lesser than the estimated amounts.

13. See pages 12 and 13.

Note that we made the implicit assumption that the insurance industry is as cost efficient on average as the broader market. While we acknowledge that this may not entirely be correct, we are interested only in the order of magnitude of the correction term. We expect cost differences driven by differences in efficiency to be an order of magnitude smaller than gross costs.

Note also that given the wide range of management costs, insurers may have incentives to model gross costs, assuming a switch from a costly asset allocation to very cheap to manage investments, well before liabilities roll off. This would reduce the present a value of overall modeled costs, at the cost of a more complex model. Implementing our measure would eliminate these incentives.

To put our estimation into the context of the industry's capital ratios, note that EIOPA's insurance statistics under the Solvency II metric, without transistionals, volatility and matching adjustments, show combined eligible own funds and solvency capital requirements for 2020 of respectively € 938 bn and € 595 bn, leading to an average Solvency ratio of 158%.

When additional cashflows in the regulatory balance sheet projection model to value liabilities are subject to profit sharing with policyholders, omitting those cashflows leads to an underestimation of the loss absorbing capacity of technical provisions (LAC TP). The LAC TP for countries with important discretionary policyholder benefits like for example Germany and France has an important impact in reducing the solvency capital requirement (SCR). With a capital ratio own funds / SCR of 200%, reducing SCR by one unit allows returning two units of capital. Hence, the impact on targeted own funds for the insurance industry may be higher than the value of the correction term.[14]

14. Assuming € 164 bn of additional cashflows are split 85/15 between policyholder and shareholder before taxes, a 30% tax rate. Assume furthermore 70% of those additional future policyholder benefits and tax payments are used to absorb losses thereby reducing SCR. Then, back-of-the-envelop, the industry's Solvency ratio would increase by 36% from

6. Concluding Remarks

In a two-period general equilibrium model, we estimated the impact on management costs on asset prices. Those assets may be listed securities, private assets, like loans, private equity, actively or passively managed portfolios, funds, etc. We argue that deducting management costs to the valuation of a portfolio based on observed or estimated prices leads to a double counting of these management costs. We propose to apply a correction term to model investment management costs in the Solvency II, SST or BSCR balance sheets. We are focused on the strictest available Solvency II metric excluding transitionals, volatility and matching adjustment (or SST, BSCR comparable metrics). An analysis with any of those measures is beyond the scope of this document. It should be noted that IFRS 17 as it is expected to be implemented would also lead to cost double counting, however to a lesser extent. [15]

A correction could hence have important consequences in terms of available capital, capital management and the investment strategy.

Perhaps the most important consequence of the status quo is the impact on the investment strategy by reducing the set of possible or affordable investment alternatives. We stress that one of the added values of a pooled investment activity such as life insurance might offer, is giving retail investors access to investments they normally cannot access fully or at all, such as for example private debt, private equity, infrastructure equity etc. Status quo modeling penalizes such investments for the wrong reasons, i.e. the fact that they are more expensive to manage. Insurers are hence pushed towards cheap to manage

158% to 194%. However, the 158% starting point is too high, as some of our correction term should already be embedded in some of the Solvency II balance sheets.

15. See in IASB (2021) explanations on *Cash flows within the contract boundary (paragraph 34)* the point B.65.(ka). The standard requires in § 33 market consistent modeling which may be achieved via "risk-neutral measurement techniques" according to B.77.

asset classes.[16] This undermines also the role the insurance sector can play in the financing of the economy as an investor which should be well positioned to support relatively complex to manage long term investments.

A correction should also mitigate capital inefficiency of traditional with profit business, which many insurers have put into run-off due to high capital consumption. In some cases it might enabling companies to redeploy capital.

Finally, eliminating double counting or not may have an impact on mergers and acquisitions for traditional life insurance portfolios. Consolidators acquiring such portfolios would typically skew the asset allocation to alternative debt, or more generally expensive to manage assets. The adverse impact of the status quo modeling on the capital needed to support the book of business would then be exacerbated.

While one may measure reasonably well the costs incurred by the investment activity[17], the data is of course not available to pinpoint perfectly the market's weighted average investment costs. However, one would expect that cost differences are an order of magnitude smaller than the absolute costs. Many institutional investors report their investment management costs, and consulting firms offer benchmarking services, through which they have of course more granular insights into the management costs of a meaningful set of investors. While this may give some guidance to determine if there is an obvious inefficiency, small cost differences might also be driven by quality differences in the approach how to invest into a certain sub-segment.

16. The appropriateness of capital requirements of those assets is beyond the scope of this paper. Out point is of course limited to the observation that such investments creates a negative cashflow strain in the insurance liability projections created by incorrect cost modeling.

17. That being said for some functions it may not always be easy to determine exactly how much of the costs are driven by investment activity and how much by other activities. For example the ALM department may drive investment decisions, but may also be involved in activities not directly related including for example liability modeling.

An approach which would factor-in management cost differences which cannot be easily attributed to higher or lower efficiency would hence risk penalizing higher diligence and hence costs, or rewarding the absence of it. This may however have to be balanced with the disciplining effect of having to evaluate the present value of unaddressed cost inefficiencies.

A pragmatic approach may be to first test if there are obvious cost differences attributable to inefficiencies. If yes, it may be reasonable to estimate their size and add the present value to the liability side of the insurance balance sheet. If the costs generated by the investment strategy seem broadly in line with the market, the most pragmatic approach may be to assume all investors costs are identical, as in formula (3.12) which then leads to investment management costs being canceled out.

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Appendix: Proof of Proposition 1

We prove the properties for $m_i = 0$. As long as the survival assumption holds the result for $m_i > 0$ then follows from (3.5). Let us here write $V_i(a, p)$ for $V_i(a, p, 0)$ and $D_i(p)$ for $D_i(p, 0)$. The map V_i has well defined partial derivative V'_{ia} with respect to a given by:

$$V'_{ia}(a, p) = \mathbf{E}\,(x - (1+\rho)p)\,U'_i\,((1+\rho)w_{i0} + a(x - (1+\rho)p))$$

Demand is given by:

$$\begin{cases} D_i(p) = 0 & \text{if } V'_{ia}(0,p) \leq 0 \\ D_i(p) = \frac{w_{i0}}{p} & \text{if } V'_{ia}(\frac{w_{i0}}{p},p) \geq 0 \\ V'_{ia}(D_i(p)) = 0 & \text{otherwise.} \end{cases}$$

For the first point, note that

$$V'_{ia}(0,p) \quad = \quad (\mathbf{E}x - (1+\rho)p)\, U'_i\left((1+\rho)(w_{ic} + w_{ia}p)\right),$$

so that $V'_{ia}(0,p)$ has the same sign as $\mathbf{E}x - (1+\rho)p$, hence the result for $m_i = 0$.

To prove the second and third points we first determine the sign of $V''_{iap}(a,p)$ for $p > 0$ and $a \in [0, \frac{w_0}{p}]$ in a similar way to Fishburn and Burr Porter (1976):

$$\begin{aligned} V''_{iap}(a,p) &= -(1+\rho)\mathbf{E}U'(w_{if}) - (x-(1+\rho)p)(w_{ia}-a)U''(w_{if}) \\ &= -(1+\rho)\mathbf{E}\left(w_{if} + (x-(1+\rho)p)(w_{ia}-a)R^r_i(w_{if})\right)\frac{U'_i(w_{if})}{w_{if}} \end{aligned}$$

with $w_{if} = w_{ic} + w_{ia}p + a(x-(1+\rho)p)$. Given that $1 \leq R^r(w_{if}) > 0$, $w_{if} + (x-(1+\rho)p)(w_{ia}-a)R^r_i(w_{if})$ is positive for $x \geq 0$: For $R^r_i(w_{if}) = 0$ its value is $w_{if} \geq 0$ a.s. and for $R^r_i(w_{if}) = 1$ its value is $w_{ic} + xw_{ia} > 0$; for $R^r_i(w_{if}) \in (0,1]$ it lies between those two values. Hence $V''_{iap}(a,p) < 0$ for all $p > 0$ and $a \in [0, \frac{w_{i0}}{p}]$.

Now for the second point. If $V'_{ia}(0,p_0) < 0$ for some p_0, then $D_i(p) = 0$ in a neighborhood of p_0, and if $V'_{ia}(\frac{w_{i0}}{p_0},p_0) > 0$ then $D_i(p) = \frac{w_{i0}}{p} = w_a + \frac{w_{ic}}{p}$ in a neighborhood of p_0. If $V'_{ia}(0,p_0) > 0$ and $V'_{ia}(\frac{w_{i0}}{p}{}_0,p_0) < 0$ then D_i is given by $V'_{ia}(D_i(p)) = 0$ in a neighborhood of p, and, by the implicit function theorem, D_i is differentiable at p_0 and $D'_i(p_0) = -\frac{V''_{iap}}{V''_{iaa}}(D_i(p_0)) < 0$. In these three cases D_i is differentiable at p_0.

At $p_0 = \frac{\mathbf{E}x - m_i}{1+\rho}$, $D_i = 0$ for $p \geq p_0$, so D_i has right-hand derivative 0 at p_0. Similarly D_i is given by $V'_{ia}(D_i(p)) = 0$ for $p \leq p_0$ close to p_0, so D_i has left-hand derivative $-\frac{V''_{iap}}{V''_{iaa}}(D_i(p_0)) < 0$ at p_0

Finally, consider p_0 such that $V'_{ia}(\frac{w_0}{p_0},p_0) = 0$. Then by the implicit function Theorem there exists a neighborhood of p_0 in which the solution $D^r_i(p)$

to $V'_{ia}(D_i(p), p) = 0$ exists, is unique, and is differentiable with derivative $-\frac{V''_{iap}}{V''_{iaa}}(D_i(p_0)) < 0$ at $p = p_0$. On this neigbhorhood, we have $D_i(p) = \min\{D_i^r(p), \frac{w_0}{p}\}$. The left-hand and right-hand derivative of D_i at p_0 exists, the right-hand derivative is the minimum of the derivatives of $D_i^r(p)$ and $\frac{w_0}{p}$ at $p = p_0$, and the left-hand derivative is the maximum of the derivatives of $D_i^r(p)$ and $\frac{w_0}{p}$ at $p = p_0$.

This shows points 2 and 3.

To prove point 4, consider $a > 0$.

We have

$$\lim_{p \to 0} V'_{ia}(a, p) = \mathbf{E}xU'_i(w_{ic}(1+\rho) + ax) > 0.$$

Choose $p_a > 0$ with $p_a < \frac{\mathbf{E}x}{1+\rho}$ such that for all positive $p < p_a$, $V'_{ia}(a, p) > 0$ and $w_{ia} + \frac{w_{ic}}{p} > a$. For such p, $D_i(p)$ is either given by $V'_{ia}(D_i(p), p) = 0$ or equal to $w_{ia} + \frac{w_{ic}}{p}$. In both cases it is greater than a. This completes point 4.