Corrections to Hierarchic Competitive Equilibria

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The proof of Theorem 5.1 in Florig (2001) has an error. The proof that $B_i \subset B_i(\mathcal{P}, w_i)$ (Step 9, page 530) needs an additional assumption on the consumption set to go through. The error lies in lines 5 - 7 of the proof of Step 9. The existence of $n_\lambda \in N$, such that for all $n \in N_{k+1}$ with $n \geq n_\lambda$,

$$p^n \cdot (z_\lambda - \omega_i - \sum_{j \in J} \theta_{ij} y_j) \leq \delta_i (1 - \|p^n\|) + \sum_{j \in J} \theta_{ij} p^n \cdot (y_j^n - y_j),$$

requires in general that for all large n, $p^n \cdot z_\lambda \leq p^n \cdot z^n$.

This need not hold. Consider for example the unit ball $I\!B$ in $I\!R^2$. Take the sequence $p^n = (\sqrt{(1 - (1/n)^2)}, 1/n)$ together with $z^n = -p^n$. Then $p^n \cdot z^n = \min p^n I\!B$. The hierarchic price would be $\mathcal{P} = (p^1, p^2)$ with $p^1 = (1, 0)$, $p^2 = (0, 1)$. So $z = \zeta = -(1, 0) = \operatorname{argmin} \mathcal{P}Y$ and for all n and all $\lambda \in [0, 1[$, $p^n \cdot z^n < p^n \cdot z_\lambda$.

There are at least two ways to correct this, depending on the additional assumption on the consumption set one imposes. Firstly, if one assumes the consumption set to be a polyhedron, then the proof of Step 9 goes through, as we will show below. Secondly, if one imposes more generally Assumption 5 in Florig (2001) to hold also for the consumption sets, the existence can be proven along slightly different lines. However, then the extra revenue of the consumers cannot be determined as beforehand by an arbitrary choice of a positive initial endowment of paper money.

Polyhedral consumption sets.

It is sufficient to prove that there exists $n_\lambda \in N$, such that for all $n \in N_{k+1}$ with $n \geq n_\lambda$, $p^n \cdot z_\lambda \leq p^n \cdot z^n$, or equivalentely $p^n \cdot (\zeta - (z + \frac{z^n - z}{1-\lambda})) \leq 0$. For all n large enough $(z + \frac{z^n - z}{1-\lambda})$ is in X_i, since X_i is a polyhedron. Moreover,

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since X_i is a polyhedron, for all large n,

$$\zeta \in \text{argmin} \sum_{\rho=s}^{\underline{r}_i} \varepsilon_\rho^n p^\rho X_i(p^1, \ldots, p^{s-1}).$$

So , if $\underline{r}_i = k$, then for all large n, $p^n \cdot (\zeta - (z + \frac{z^n - z}{1-\lambda})) \leq 0$.

Otherwise, since ζ may have been chose such that

$$\{p^s, \ldots, p^{\underline{r}_i+1}\}\zeta < \min \{p^s, \ldots, p^{\underline{r}_i+1}\}\{z, z^1, \ldots, z^n, \ldots\},$$

there exists for a subsequence of z^n and $r' \in \{s, \ldots, \underline{r}_i + 1\}$ such that

$$p^r \cdot \zeta \leq \min p^r\{z, z^1, \ldots, z^n, \ldots\},$$

for all $r \leq r'$ with a strict imequality for $r = r'$. Therefore

$$\sum_{\rho=s}^{r'} \varepsilon_\rho^n p^\rho \cdot (\zeta - (z + \frac{z^n - z}{1 - \lambda})) < -M\varepsilon_{r'}^n,$$

for some $M > 0$ and all large enough n. Of course, we also have some $m > 0$ such that for all large n,

$$\sum_{\rho=r'+1}^{k} \varepsilon_\rho^n p^\rho \cdot (\zeta - (z + \frac{z^n - z}{1 - \lambda})) \leq m\varepsilon_{r'+1}^n.$$

Therefore, we have for all large enough n,

$$p^n \cdot (\zeta - (z + \frac{z^n - z}{1 - \lambda})) < 0.$$

Consumption sets satisfying Assumption 5.

In this case, we need to proceed differently. Note

$$w_i^n = p^n \cdot (\omega_i + \sum_{j \in J} \theta_{ij} \tilde{S}_j(p^n)) + \delta_i(1- \parallel p^n \parallel).$$

Let $w_i = \sup \mathcal{P}B_i$. The inclusion $B_i \subset B_i(\mathcal{P}, w_i)$ is trivial. In order to show that $B_i(\mathcal{P}, w_i) \subset B_i$, let $z \in B_i(\mathcal{P}, w_i)$ such that $\mathcal{P}z \leq w_i$. There exists $\bar{z} \in B_i$ such that $\mathcal{P}z \leq \mathcal{P}\bar{z}$ and there exists z^n converging to $\bar{z}$ with $z^n \in B_i(p^n)$. First, note that for all large enough n, $p^n \cdot (z - \bar{z}) \leq 0$. Moreover, by Assumption 5, there exists $\lambda^n \in [0, 1[$ converging to 1 such that for all large n, $z^n + \lambda^n(z - \bar{z}) \in X_i$. Since

$$p^n \cdot (z^n + \lambda^n(z - \bar{z})) \leq w_i^n + \lambda^n p^n \cdot (z - \bar{z}) \leq w_i^n$$

and $(z^n + \lambda^n(z - \bar{z}))$ converges to z, we have $z \in B_i$.

Now, by the closure of B_i, we have $B_i(\mathcal{P}, w_i) \subset B_i$.

References

Florig, M., 2001, Hierarchic Competitive Equilibria, *Journal of Mathematical Economics*, 35(4), 515-546.